ANALYSIS OF MULTI-LAYER COMPOSITE SIMPLY SUPPORTED BEAM UNDER BLAST LOADING

Dr. Khaleel I. Aziz   **Mr. Ahmed T. Al – Ejbari**
University of Anbar   **University of Anbar**

ABSTRACT

In this study an attempt is made to derive governing equations satisfying equilibrium and compatibility, for multi-layer composite simply supported beam under blast loading , for linear material and shear connector behavior in which the slip (horizontal displacement) and uplift force (vertical displacement) are taken into consideration. The analysis is based on an approach presented by Roberts, which takes into consideration horizontal and vertical displacements in interfaces. The model consists of a simply supported beam with three layers having a cross-sectional area ,different dimensions and material properties. The analysis led to a set of six differential equations containing derivatives of the fourth and third order. The blast loading was considered as a function of time. Explosions have different effects including blast, penetrations and fragmentation. The blast is the main effect which hits the structure in short duration. Multi –layer composite construction is the best type of constructions to resist the blast loading ; according to this , multi-layer composite construction is used for air-craft and marine industries. Analysis of composite beam under blast load , taking in consideration vertical and horizontal displacements, leads to six differential equations , the load is taken as a function of time.

KEYWORDS: multi-layer, composite , beam, blast, simply supported

1.INTRODUCTION

An explosive is a mixture or chemical compound which, under the influence of heat or shock, undergoes a chemical reaction of the following types:

1.A considerable amount of energy is liberated.

2.The explosive is converted into gas which is at high temperature and under high pressure.

3.The reaction needs no support from the outside , if started under certain conditions , it will spread through the entire explosive[1]

Blast

When detonation of a charge takes place in the air, the case of charge bursts and the surrounding air is compressed , so much heat is created in the air that it becomes luminous. The outer surface of luminous region is called the flame front. After the case of the charge breaks , the gaseous products of the explosion burst forth and continue to expand. The initial velocity of expansion of these gases varies from 6000 to 30000 feet per second. The gases cool rapidly and lose most of their velocity after they have gone a distance 40 to 50 times the diameter of the charge. The variations of pressure of these gases are carried into the surrounding air where they influence the form of the blast wave.

The blast wave from an exploding charge is characterized by an immediate rise in pressure from atmospheric to the peak pressure of the blast wave followed by a rapid decrease to sub-atmospheric pressure and then a slower increase to atmospheric value. Thus the blast wave has two phases : the abrupt rise of pressure followed by a decrease to atmospheric is called the positive or pressure phase, and the decrease of pressure to sub-atmospheric followed by a return to atmospheric is called negative or suction phase, as shown in Fig.(1).

The area under the positive phase of the pressure-time curve is called the positive impulse of the blast, the area under the remaining portion is called the negative one. The instantaneous rise in pressure results from there being a region or surface around the detonation of a bomb in front of which the air is undisturbed and behind which the air is under great pressure. The surface is called the shock front of the blast wave. In air or compressible fluids, high pressure creates a shock front because they heat the medium and expand rapidly.

The physical characteristics of the shock wave may be defined in terms of peak pressure and impulse of various distance.

The peak pressure is maximum during the initial phase. Peak pressure from bombs , cased and uncased charges , may be determined from the following formula[1]:

$$p_s = \frac{4120}{Z^3} - \frac{105}{Z^2} + \frac{39.5}{Z}$$

(1)

Another formula suggested by Birmam and Bert is[2]:

$$p_s = p_{so}(1 - \frac{t}{t_0})e^{\frac{t}{t_0}}$$

(2)

2. COMPOSITE CONSTRUCTION

Composite construction has been widely used in building construction. A perfect connection between the components of composite elements exists only theoretically. Experimental investigation has shown that significant slip occurs at the interface between these components, even when a large number of connectors are provided. The modification in the behavior of a composite beam by the presence of slip was illustrated by analysis conducted by many researchers. These analyses led to differential equations (number of which depends on the degree of freedom) that are to be solved fresh for each type of loading and the variation in dimensions or properties of beams. The first interaction theory that takes account of slip effects was initially formulated by Newmark et al.[4], based on elastic analysis of composite beams assuming linear material and shear connector behavior. Adekola [5] presented a different model based on interaction theory, which takes account of slip, uplift and friction effect. Using the same element presented by Newmark et al., Johnson [6] in 1975 proposed a partial interaction theory for simply supported beams, in which the analysis was based on elastic theory. The composite beam was assumed to have linear elastic materials. Roberts [3] presented an approach for the analysis of composite beam with partial interaction, in which the basic equilibrium and compatibility equations were formulated in terms of four independent variables, i.e. the axial displacements of the concrete and steel and the deflections of the two layers. Linear elastic materials and shear connector behavior were assumed with the concrete remaining uncracked, and both the slip and separation at the interface were incorporated.
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3. MATHEMATICAL MODEL

Interaction theories which consider the slip, were formulated by many researchers. Newmark et al. [4] presented one of the earliest linear analyses of composite beams, incorporating the influence of slip. A recent approach has been presented by Roberts[15] in which the basic equilibrium and compatibility equations are derived in terms of displacements and solved simultaneously using a finite difference representing the various derivatives. Al-Amery [16] presented a new approach, considering non-linear materials and shear connector behavior.

In this study, the model, consists of three different layers, different materials and shear and normal stiffness. The analysis leads to a set of six basic equilibrium and compatibility equations that were formulated in terms of displacements (horizontal and vertical) of each layer. These differential equations were expressed in finite difference form, and the resulting simultaneous algebraic equations were solved numerically.

For composite elements to act as a single structural unit, an efficient connection must be provided, which serves to transfer shear and normal forces and prevent significant relative movement between the layers, where the connectors are flexible, the slip and separation, may occur at the interfaces.

3.1 Assumptions

The basic assumptions of conventional beam theory were used in which plane sections are assumed to remain plane. Also, the connection was assumed to have negligible thickness and posses finite normal and tangential stiffness.

3.2 Equilibrium

An element of a composite of three layers, length δx, shown in Fig.(2), is subjected to moments, M, shear forces, V, and axial forces, F, subscripts a, b, and c denotes three layers from upper to lower layer, and the local x-z axes pass through the centroids of the materials. If the beam is subjected to blast loading according to Eq.(1) plus distributed load ρ per unit length, vertical equilibrium of the whole element gives:

$$\delta V_a + \delta V_b + \delta V_c = \rho \delta x + P_s \delta x \tag{3}$$

Dividing equation (3) by δx and taking a limit as δx tends to zero gives:

$$V_{a,x} + V_{b,x} + V_{c,x} = \rho + P_s \tag{4}$$

In which subscript x denote differentiation, for live load only ρ_i, for live load and dead load, ρ is equal to:

$$\rho = \rho_i + \rho_a + \rho_b + \rho_c \tag{5}$$

In which ρ_a, ρ_b and ρ_c are the distributed self-weight of the three layers. Loads due to the removal of props used during construction should be considered as live loads.
Taking moments about the origin of coordinates in the upper layer gives:

$$\delta M_a + \delta M_b + \delta M_c = (V_a + V_b + V_c).\delta x + (\delta V_a + \delta V_b + \delta V_c).\frac{\delta x}{2} + \delta F_b.d_1 + \delta F_c.(d_1 + d_2) \tag{6}$$

In which d_1 and d_2 are the distance between the centroids of the upper and middle cross section and middle and lower cross section respectively.
After neglecting the second order terms and dividing by δx Eq. (6) becomes:
$$M_{a,x} + M_{b,x} + M_{c,x} = V_a + V_b + V_c + F_{b,x}.d_1 + F_{c,x}.(d_1 + d_2) \tag{7}$$
Differentiating Eq. (7) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} = V_{a,x} + V_{b,x} + V_{c,x} + F_{b,xx}.d_1 + F_{c,xx}.(d_1 + d_2) \tag{8}$$

Substituting Eq.(4) into Eq.(8) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} - F_{b,xx}.d_1 - F_{c,xx}.(d_1 + d_2) = \rho + P_s \tag{9}$$

Taking moments about the origin of coordinate in the middle layer gives:

$$\delta M_a + \delta M_b + \delta M_c = (V_a + V_b + V_c).\delta x + (\delta V_a + \delta V_b + \delta V_c).\frac{\delta x}{2} + \delta F_a.d_1 - \delta F_c.d_2 \tag{10}$$

After neglecting the second order terms and dividing by δx Eq. (10) becomes:

$$M_{a,x} + M_{b,x} + M_{c,x} = V_a + V_b + V_c - F_{a,x}.d_1 + F_{c,x}.d_2 \tag{11}$$

Differentiating Eq.(11) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} = V_{a,x} + V_{b,x} + V_{c,x} - F_{a,xx}.d_1 + F_{c,xx}.d_2 \tag{12}$$

Substituting Eq.(4) into (12) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + F_{a,xx}.d_1 - F_{c,xx}.d_2 = \rho + P_s \tag{13}$$

For equilibrium of the composite element, shown in **Fig. (2)**, in the x-direction givses:

$$(\delta F_a + F_a) + (\delta F_b + F_b) + (\delta F_c + F_c) - (F_a + F_b + F_c) = 0 \tag{14}$$

$$\delta F_a + \delta F_b + \delta F_c = 0 \tag{15}$$

Dividing Eq.(15) by δx gives:

$$F_{a,x} + F_{b,x} + F_{c,x} = 0 \tag{16}$$

Eqs. (9), (13), and (16) are the three basic equilibrium equations required for the complete solution.

3.3 Compatibility

Assuming plane sections within each material remain plane, the total displacement of the upper layer in the x-direction at the interface, denoted by U_{ati}, is given by:

$$U_{ati} = u_a - z_{ai}.w_{a,x} \tag{17}$$

In which z_{ai} is the z-coordinate of the interface relative to the local x-z axes and, u_a and w_a are the displacements of the upper layer in the x and z direction. Similarly for the other two layers:

$$U_{bti} = u_b - z_{bi}.w_{b,x} \tag{18}$$

$$U_{cti} = u_c - z_{ci}.w_{c,x} \tag{19}$$

The slip, U_{ab}, at the interface between the first two layers is denoted as the relative displacement in the x-direction of initially adjacent particles, as shown in Fig.(2). Hence:

$$U_{ab} = U_{ati} - U_{bti} \tag{20}$$

And between the lower two layers:

$$U_{bc} = U_{bti} - U_{cti} \tag{21}$$

Combining Equations (17), (18), (19), (20) and (21) gives:

$$U_{ab} = (u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x}) \tag{22}$$

$$U_{bc} = (u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x}) \tag{23}$$

If the shear stiffness of the joint per unit length between the upper two layers, denoted by k_{s1}, the shear force per unit length at the interface q_1 is given by:

$$q_1 = k_{s1}.U_{ab} \tag{24}$$

The shear force per unit length at the interface of the lower layers, q_2, is given by:

$$q_2 = k_{s2}.U_{bc} \tag{25}$$

Considering the equilibrium of the upper layer in the x-direction gives:

$$F_{a,x} = q_1 = k_{s1}.U_{ab} \tag{26}$$

Considering the equilibrium of the middle layer in the x-direction gives:

$$F_{b,x} = q_2 - q_1 \tag{27}$$

$$F_{b,x} = k_{s2}.U_{bc} - k_{s1}.U_{ab} \tag{28}$$

$$F_{a,x} + F_{b,x} = q_2 = k_{s2}.U_{bc} \tag{29}$$

Substituting for U_{ab} from Eq. (22) into (26) gives:

$$F_{a,x} - k_{s1}[(u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x})] = 0 \tag{30}$$

Substituting for U_{bc} from Eq. (23) into (29) gives:

$$F_{a,x} + F_{b,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0 \tag{31}$$

The separation at the interface between the first upper layers, w_{ba} is the relative displacement in the z-direction of initially adjacent, as shown in Fig.(2) is given by:

$$W_{ba} = w_b - w_a \tag{32}$$

The separation at the interface between the two lower layers, w_{cb} is given by:

$$W_{cb} = w_c - w_b \tag{33}$$

If P_1 denotes the normal force per unit length at the interface equilibrium at the two upper layers element in the z-direction it is given by:

$$V_{a,x} = \rho_i + \rho_a + P_1 + P_s \tag{34}$$

If P_2 denotes the normal force per unit length at the interface equilibrium at the lower two layers element in the z-direction is given by:

$$V_{b,x} = P_2 - P_1 + \rho_b \tag{35}$$

$$V_{a,x} + V_{b,x} = P_2 + \rho_b + \rho_a + \rho_i + P_s \tag{36}$$

Consider the moment equilibrium of the upper layer element about the origin of coordinates which gives:

$$V_a = M_{a,x} + q_1.z_{ai} \tag{37}$$

Consider the moment equilibrium of the second layer element about the origin of coordinates which gives:

$$V_b = M_{b,x} + q_2.z_{bi} - q_1.z_{bi} \tag{38}$$

Differentiating Eqs. (37) and (38) gives:

$$V_{a,x} = M_{a,xx} + q_{1,x}.z_{ai} \tag{39}$$

$$V_{b,x} = M_{b,xx} + q_{2,x}.z_{bi} - q_{1,x}.z_{bi} = M_{b,xx} + F_{b,xx}.z_{bi} \tag{40}$$

Differentiating Eqs. (26) and (29) with respect to x gives:

$$F_{a,x} = q_{1,x} \tag{41}$$

$$F_{a,xx} + F_{b,xx} = q_{2,x} \tag{42}$$

Substituting Eqs. (34) and (36) into (39) and (40) gives:

$$P_1 = M_{a,xx} + q_{1,x}.z_{ai} - (\rho_i + P_s + \rho_a) \tag{43}$$

$$V_{a,x} + V_{b,x} = M_{a,xx} + M_{b,xx} + q_{2,x}.z_{bi} - q_{1,x}.z_{bi} + q_{1,x}.z_{ai} \tag{44}$$

$$P_2 = M_{a,xx} + M_{b,xx} + q_{2,x}.z_{bi} - q_{1,x}.z_{bi} + q_{1,x}.z_{ai} - (\rho_i + \rho_a + \rho_b) - P_s \tag{45}$$

Substituting Eq. (41) into (42) gives:

$$P_1 = M_{a,xx} + F_{a,xx}.z_{ai} - (\rho_i + P_s + \rho_a) \tag{46}$$

Substituting Eq. (42) into (45) gives:
$$P_2 = M_{a,xx} + M_{b,xx} + F_{a,xx}.z_{ai} + F_{b,xx}.z_{b,xx} - (\rho_i + P_s + \rho_a + \rho_b) \tag{47}$$
If the normal stiffness of the joint per unit length between the upper layers, is denoted by k_{n1}, then:
$$P_1 = k_{n1}.W_{ba} = k_{n1}.(w_b - w_a) \tag{48}$$

If the normal stiffness of the joint per unit length between the lower layers, is denoted by k_{n2}, then:

$$P_2 = k_{n2}.W_{cb} = k_{n2}.(w_c - w_b) \tag{49}$$

Substituting Eq. (48) into Eq. (46) gives:
$$M_{a,xx} + F_{a,xx}.z_{ai} - k_{n1}(w_b - w_a) = \rho_i + \rho_a + P_s \tag{50}$$

Substituting Eq. (49) into Eq. (47) and combining with Eq. (49) gives:

$$M_{b,xx} + F_{b,xx}..z_{bi} - k_{n2}(w_c - w_b) + k_{n1}(w_b - w_a) = \rho_b \tag{51}$$

Eqs.(30), (31) and (51) are the three basic compatibility equations required for a complete solution.

BASIC DIFFERENTIAL EQUATIONS
From the analytical model, the six independent differential equations (equilibrium and compatibility), may be expressed in terms of displacement variables, u_a, w_w, u_b, w_b, u_c and w_c as follows :Assuming plane sections within each material remain plane, the axial strain ε can be expressed in terms of displacements u, w relative to the local x and z –axes, which are assumed to pass through the centroid of the three materials. Hence:

$$\varepsilon_a = U_{at,x} = U_{a,x} - z_a.w_{a,xx} \tag{52}$$
$$\varepsilon_b = U_{bt,x} = U_{b,x} - z_b.w_{b,xx} \tag{53}$$
$$\varepsilon_c = U_{ct,x} = U_{c,x} - z_c.w_{c,xx} \tag{54}$$

In which subscripts a, b and c denotes the layers, subscripts x denotes differentiation and z the distance form the origin of coordinates to the limits of the layers.

Stresses, now can be related to strain via the materials properties E_a, E_b and E_c. For linear elastic materials E_a, E_b and E_c are constants, but for nonlinear elastic and elasto-plastic materials, E_a, E_b and E_c are functions of strain.

The free strain due to shrinkage, temperature ,etc, are denoted by ε_f, while the strain induced during the construction sequence, are denoted by ε_r. Hence, if u and w are assumed exclude the displacements corresponding, to ε_f and ε_r, the stresses in the layers are given by:

$$\sigma_a = E_a(u_{a,x} - z_a.w_{a,xx} + \varepsilon_{ra} - \varepsilon_{fa}) \tag{55}$$

$$\sigma_b = E_b(u_{b,x} - z_b.w_{b,xx} + \varepsilon_b - \varepsilon_{fb}) \tag{56}$$

$$\sigma_c = E_c(u_{c,x} - z_c.w_{c,xx} + \varepsilon_{rc} - \varepsilon_{fc}) \tag{57}$$

The axial forces, F_a, F_b and F_c, and moments M_a, M_b, and M_c are obtained by integrating the stresses, multiplying by the appropriate lever arms, z_a, z_b and z_c, in the case of moments over the cross section area of each layer which denoted by A_a, A_b and A_c. Hence:

$$F_a = \int \sigma_a.dA_a \tag{58}$$

$$F_b = \int \sigma_b.dA_b \tag{59}$$

$$F_c = \int \sigma_c.dA_c \tag{60}$$

$$M_a = -\int \sigma_a.z_a.dA_a \tag{61}$$

$$M_b = -\int \sigma_b.z_b.dA_b \tag{62}$$

$$M_c = -\int \sigma_c.z_c.dA_c \tag{63}$$

Substituting Eqs. (55), (56), (57) into Eqs. (58) to (63) gives:

$$F_a = \int E_a.(u_{a,x} - z_a.w_{a,xx} + \varepsilon_{ra} - \varepsilon_{fa})dA_a \tag{64}$$

$$F_b = \int E_b.(u_{b,x} - z_b.w_{b,xx} + \varepsilon_{rb} - \varepsilon_{fb})dA_b \tag{65}$$

$$F_c = \int E_c.(u_{c,x} - z_c.w_{c,xx} + \varepsilon_{rc} - \varepsilon_{fc})dA_c \tag{66}$$

$$M_a = -\int E_a.(u_{a,x} - z_a.w_{a,xx} + \varepsilon_{ra} - \varepsilon_{fa}).z_a.dA_a \tag{67}$$

$$M_b = -\int E_b.(u_{b,x} - z_b.w_{b,xx} + \varepsilon_{rb} - \varepsilon_{fb}).z_b.dA_b \tag{68}$$

$$M_c = -\int E_c.(u_{c,x} - z_c.w_{c,xx} + \varepsilon_{rc} - \varepsilon_{fc}).z_c.dA_c \tag{69}$$

IF E_a, E_b, and E_c are constants, integration of Eqs. (64) to (69) gives:

$$F_a = E_a.A_a.u_{a,x} + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa}) \tag{70}$$

$$F_b = E_b.A_b.u_{b,x} + E_b.(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb}) \tag{71}$$

$$F_c = E_c.A_c.u_{c,x} + E_c.(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc}) \tag{72}$$

$$M_a = E_a.I_a.w_{a,xx} \tag{73}$$

$$M_b = E_b.I_b.w_{b,xx} \tag{74}$$

$$M_c = E_c.I_c.w_{c,xx} \tag{75}$$

in which, I_a, I_b, and I_c are the second moments of area for the layers and $\bar{\varepsilon}$ is the integration of the strain function over the cross section area of the corresponding materials. The following are the six governing equations derived for the three layers composite beam:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} - F_{b,xx}.d_1 - F_{c,xx}.(d_1 + d_2) = \rho + P_s \tag{76}$$

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + F_{a,xx}.d_1 - F_{c,xx}.d_2 = \rho + P_s \tag{77}$$

$$F_{a,x} + F_{b,x} + F_{c,x} = 0 \tag{78}$$

$$F_{a,x} - k_{s1}[(u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x})] = 0 \tag{79}$$

$$F_{a,x} + F_{b,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0 \tag{80}$$

$$M_{b,xx} + F_{b,xx}..z_{bi} - k_{n2}(w_c - w_b) + k_{n1}(w_b - w_a) = \rho_b \tag{81}$$

After substituting Eqs. from (70) to (75) into Eqs. from (76) to (81) gives:

$$E_a.I_a.w_{a,xxxx} + E_b.I_b.w_{b,xxxx} + E_c.I_c.w_{c,xxxx} - E_b.A_b.d_1.u_{b,xxx} - E_b(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,xx}.d_1 - (d_1 + d_2).E_c.A_c.u_{c,xxx}$$
$$- E_c.(d_1 + d_2)(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc})_{,xx} = \rho + P_s \tag{82}$$

$$E_a.I_a.w_{a,xxxx} + E_b.I_b.w_{b,xxxx} + E_c.I_c.w_{c,xxxx} + E_a.A_a.d_1.u_{a,xxx} + E_a(\bar{\varepsilon}_a - \bar{\varepsilon}_a)_{,xx}.d_1$$
$$- d_2.E_c.A_c.u_{c,xxx} - E_c.d_{2.}(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc})_{,xx} = \rho + P_s \tag{83}$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,x} + E_b.A_b.u_{b,xx} + E_b(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,x} + E_c.A_c.u_{c,xx} + E_c.(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc})_{,x} = 0 \tag{84}$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0 \tag{85}$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,x} + E_b.A_b.u_{b,xx} + E_b(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0 \tag{86}$$

$$E_b.I_b.w_{b,xxxx} + E_b.A_b.u_{b,xx}.z_{bi} - k_{n2}.(w_c - w_b) + k_{n1}(w_b - w_a) = \rho_b \tag{87}$$

5.NUMERICAL SOLUTIONS

Eqs. (82) o (87) contain derivatives of third order in u and fourth order in w, which can be expressed in finite (central) difference form using five node points as shown in Fig.(3), for example, the derivatives of w at node n can be expressed as:

$$w_{n,x} = \frac{w_{n+1} - w_{n-1}}{2.\Delta x} \tag{88}$$

$$w_{n,xx} = \frac{w_{n+1} - 2.w_n + w_{n-1}}{\Delta x^2} \tag{89}$$

$$w_{n,xxx} = \frac{w_{n+2} - 2.w_{n+1} + 2.w_{n-1} - w_{n-2}}{2.\Delta x^3} \tag{90}$$

$$w_{n,xxxx} = \frac{w_{n+2} - 4.w_{n+1} + 6.w_n - 4.w_{n-1} + w_{n-2}}{\Delta x^4} \tag{91}$$

in which Δx is the spacing of nodes.

After expressing Eqs. (82) to (87) in finite difference form, the complete solution system of algebraic equations, six degree of freedom per node, can be solved for the unknown displacements at the nodes, and its two external nodes are required at each end of the beam. In general, since the model is done for uniform distribution load and to specify the boundary conditions, the point load P can be idealized as a uniform distribution load $\rho = P/\Delta x$, applied over a single node spacing.

BOUNDARY CONDITIONS

Solution of the resulting set of algebraic equations requires the specification of boundary conditions. In general, since the equations contains a derivative of fourth order they require two external nodes to specify the boundary conditions at each end. However, if each external node is assigned six degrees of freedom per node, then twelve boundary conditions required for each end of the beam must be specified:

$$w_c = 0 \qquad \text{at} \quad x = 0 \quad \text{when } x = L \tag{92}$$

$$w_{a,xx} = 0 \qquad \text{at} \quad x = 0 \quad \text{when } x = L \tag{93}$$

$$w_{b,xx} = 0 \qquad \text{at} \quad x = 0 \quad \text{when } x = L \tag{94}$$

$$w_{c,xx} = 0 \qquad \text{at} \quad x = 0 \quad \text{when } x = L \tag{95}$$

$$u_c = 0 \qquad \text{at} \quad x = 0 \tag{96}$$

$$u_{c,x} = 0 \qquad \text{at} \quad x = 0 \tag{97}$$

$$u_{a,x} = 0 \qquad \text{at} \quad x = 0 \quad \text{when} \quad x = L \tag{98}$$

$$u_{b,x} = 0 \qquad \text{at} \quad x = 0 \quad \text{when} \quad x = L \tag{99}$$

$$V_a + V_b + V_c = R_r \qquad \text{at} \quad x = 0 \tag{100}$$

$$V_a + V_b + V_c = R_l \qquad \text{at} \quad x = L \tag{101}$$

$$u_{a,xxxx} = 0 \qquad \text{at} \quad x = 0 \quad \text{when} \quad x = L \tag{102}$$

$$u_{b,xxxx} = 0 \qquad \text{at} \quad x = 0 \quad \text{when} \quad x = L \tag{103}$$

$$u_{c,xxxx} = 0 \qquad \text{at} \quad x = 0 \quad \text{when} \quad x = L \tag{104}$$

$$U_{ab,x} = 0 \qquad \text{at} \quad x = 0 \quad \text{when} \quad x = L \tag{105}$$

in which, R_r and R_l are the reactions at the supports, Eqs.(100) and (101) express the conditions that the sum of the shear forces in the layers are equal to the support reaction R_r and R_l. And V_a, V_b and V_c can be expressed in terms of displacements derivatives as follows: consider moment equilibrium of the upper layer about the origin of coordinate, Fig.(2), gives:

$$V_a = M_{a,x} + q_1.z_{ai} \tag{106}$$

Substituting for q_1 from equation (26) into equation (106) gives:

$$V_a = M_{a,x} + F_{a,x}.z_{ai} \tag{107}$$

Similarly, for the middle layer:

$$V_b = M_{b,x} + F_{b,x}.z_{bi} \tag{108}$$

And for the lower layer:

$$V_c = M_{c,x} + F_{c,x}.z_{ci} \tag{109}$$

Substituting the forces and moments in terms of derivatives from Eqs.(70) to (75) into Eqs. (107) to (109), gives:

$$V_a = E_a.I_a.w_{a,xxx} + E_a.A_a.z_{ai} + E_a.z_{ai}.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,x} \tag{110}$$
$$V_b = E_b.I_b.w_{b,xxx} + E_b.A_b.z_{bi} + E_b.z_{bi}.(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,x} \tag{111}$$
$$V_c = E_c.I_c.w_{c,xxx} + E_c.A_c.z_{ci} + E_c.z_{ci}.(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc})_{,x} \tag{112}$$

And for the latest boundary conditions, substituting Eq. (22) into (105) gives:

$$U_{ab,x} = (u_{a,x} - z_{ai}.w_{a,xx}) - (u_{b,x} - z_{bi}.w_{b,xx}) \tag{113}$$

But Eq. (113) into a finite difference forms, gives:

$$\frac{1}{\Delta x^2}(u_{a_{n+1}} - 2.u_{a_n} + u_{a_{n-1}}) - \frac{z}{2.\Delta x^3}(.w_{a_{n+2}} - 2.w_{a_{n-1}} + 2.w_{a_{n-1}} - w_{a_{n-2}}) - \frac{1}{\Delta x^2}(u_{b_{n+1}} - 2.u_{b_n} + u_{b_{n-1}})$$
$$+ \frac{1}{2.\Delta x^3}(.w_{b_{n+2}} - 2.w_{b_{n-1}} + 2.w_{b_{n-1}} - w_{b_{n-2}}) = 0$$

$$\tag{114}$$

The details about the six governing equations and the twenty four boundary conditions, after substituting the finite difference form,

It is noted that the free strain due to shrinkage and temperature, etc and strain induced during construction sequence are neglected.

The main equations after substituting the finite difference form become:

$$\frac{E_a.I_a}{\Delta x^4}(w_{a_{n+2w}} - 4.w_{a_{n+1}} + 6.w_{a_n} - 4.w_{a_{n-1}} + w_{a_{n-2}}) + \frac{E_b.I_b}{\Delta x^4}(w_{b_{n+2}} - 4.w_{b_{+n1}} + 6.w_{b_n}$$

$$- 4.w_{b_{n-1}} + w_{b_{n-2}}) + \frac{E_c.I_c}{\Delta x^4}(w_{c_{n+2}} - 4.w_{c_{n+1}} + 6.w_{c_n} - 4.w_{c_{n-1}} + w_{c_{n-2}}) - \frac{E_b.A_b.d_1}{2.\Delta x^3}(u_{b_{n+2}}$$

$$- 2.u_{b_{n+1}} + 2.u_{b_{n-1}} - u_{b_{n-2}}) - E_b.d_1.(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,xx} - \frac{E_c.A_c(d_1 + d_2)}{2.\Delta x^3}(u_{c_{n+2}} - 2.u_{c_{n+1}}$$

$$+ 2.u_{c_{n-1}} - u_{c_{n-2}}) - E_c.(d_1 + d_2).(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc}) = \rho + P_s$$

$$(115)$$

$$\frac{E_a.I_a}{\Delta x^4}(w_{a_{n+2w}} - 4.w_{a_{n+1}} + 6.w_{a_n} - 4.w_{a_{n-1}} + w_{a_{n-2}}) + \frac{E_b.I_b}{\Delta x^4}(w_{b_{n+2}} - 4.w_{b_{+n1}} + 6.w_{b_n}$$

$$- 4.w_{b_{n-1}} + w_{b_{n-2}}) + \frac{E_c.I_c}{\Delta x^4}(w_{c_{n+2}} - 4.w_{c_{n+1}} + 6.w_{c_n} - 4.w_{c_{n-1}} + w_{c_{n-2}}) + \frac{E_a.A_a.d_1}{2.\Delta x^3}(u_{a_{n+2}}$$

$$- 2.u_{a_{n+1}} + 2.u_{a_{n-1}} - u_{a_{n-2}}) + E_a.d_1.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,xx} - \frac{E_c.A_c d_2}{2.\Delta x^3}(u_{c_{n+2}} - 2.u_{c_{n+1}}$$

$$+ 2.u_{c_{n-1}} - u_{c_{n-2}}) - E_c.d_2.(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc}) = \rho + P_s$$

$$(116)$$

$$\frac{E_a.A_a}{\Delta x^2}(u_{a_{n+1}} - 2.u_{a_n} + u_{a_{n-1}}) + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,xx} + \frac{E_b.A_b}{\Delta x^2}(u_{b_{n+1}} - 2.u_{b_n} + u_{b_{n-1}})$$

$$+ E_b.(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,x} + \frac{E_c.A_c}{.\Delta x^2}(u_{c_{n+1}} - 2.u_{c_n} + u_{c_{n-1}}) - E_c.(\bar{\varepsilon}_{rc} - \bar{\varepsilon}_{fc}) = 0$$

$$(117)$$

$$\frac{E_a.A_a}{\Delta x^2}(u_{a_{n+1}} - 2.u_{a_n} + u_{a_{n-1}}) + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,x} - k_{s1}.u_{a_n} + \frac{k_{s1}.z_{ai}}{2.\Delta x}(w_{a_{n+1}} - w_{a_{n-1}}) +$$

$$+ k_{s1}.u_{b_n} - \frac{k_{s1}.z_{b1}}{2..\Delta x}(w_{b_{n+1}} - w_{b_{n-1}}) = 0$$

$$(118)$$

$$\frac{E_a.A_a}{\Delta x^2}(u_{a_{n+1}} - 2.u_{a_n} + u_{a_{n-1}}) + E_a.(\bar{\varepsilon}_{ra} - \bar{\varepsilon}_{fa})_{,xx} + \frac{E_b.A_b}{\Delta x^2}(u_{b_{n+1}} - 2.u_{b_n} + u_{b_{n-1}})$$

$$+ E_b.(\bar{\varepsilon}_{rb} - \bar{\varepsilon}_{fb})_{,x} - k_{s2}.u_{b_n} + \frac{k_{s2}.z_{bi}}{2.\Delta x}(w_{b_{n+1}} - w_{b_{n-1}}) + k_{s2}.u_{c_n} - \frac{k_{s2}.z_{ci}}{2.\Delta x}.(w_{c_{n+1}} - w_{c_{n-1}}) = 0$$

$$(119)$$

$$\frac{E_b.I_b}{\Delta x^4}.[w_{b_{n+2}} - 4.w_{b_{+n1}} + 6.w_{b_n} - 4.w_{b_{n-1}} + w_{b_{n-2}}] - \frac{E_b.A_b.z_{bi}}{2.\Delta x^3}[u_{b_{n+2}} - 2.u_{b_{n+1}} + 2.u_{b_{n-1}} - u_{b_{n-2}}]$$

$$- k_{n2}.(w_{c_n} - w_{b_n}) + k_{n2}.(w_{b_n} - w_{a_n}) = \rho_b$$

$$(120)$$

4. APPLICATIONS

The developed analysis presented above takes into consideration interlayer slip and has been applied to previous examples tested by other researchers.

A computer program has been written to solve the set of differential equations using finite difference method . A case study is considered to study the effect of blast loads on composite multi-layer simply supported beam.

5. ANALYSIS AND DISCUSSION

As noted previously, the method of analysis developed was investigated under a certain case study. An analytical study is carried out for a multi-layer composite simply supported beam, the dimensions of the beam are shown in Fig.(4). The loads and structural details are shown in Table(1) based on formula according to TM-855-1to calculate the peak over pressure and duration. The material properties of steel, concrete and shear connections are presented in Table(2).

Fig.(5) shows the time- deflection relationship for the three layer simply supported beam during short time of exposure to blast load, the figure shows a difference in values of deflection for the layers. The blast load is applied directly to the upper layer and the deflection is large compared with other layers; in fact the deflection of the interior layer is small and can be used to protect the inside persons and equipment from other explosions effects such as penetration and crater.

Fig.(6) shows the time- deflection relationship for three layer simply supported beam during short time of exposure to blast load, the figure shows a difference in values of deflection for the layers.

6. CONCLUSIONS

The main conclusions that can summarized is as follow:

1 - Explosions have different effects including blast, penetrations and fragmentation. The blast is the main effect which hits the structure in short duration. Multi –layer composite construction is the best type of constructions to resist the blast loading ; according to this , multi-layer composite construction is used for air-craft and marine industries.

2 - A special case of multi-layered composite construction, is steel - concrete -steel sandwich beams (SCSS) or double skin composite construction (DSC). This is a relatively new and innovative form of construction, consisting mainly of a layer of plain concrete sandwiched between two layers of relatively thin steel plates connected to the concrete by welded stud connectors.

3 - Analysis of composite beam under blast load , taking in consideration vertical and horizontal displacements, leads to six differential equations , the load is taken as a function of time.

4 - A case study was considered to calculate the slip and deflection for the three layer composite simply supported beam.

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NOTATION

a, b, and c= Subscript denotes different layers.

A_a , A_b and A_c = Cross-sectional area of different layers.

A = Effective width of concrete slab.

d_1 and d_2 =Distance between the centroids of successive layers.

E_1 = Modulus of elasticity of concrete.

E_2 = Modulus of elasticity of steel.

E_a , E_b and E_c =Modulus of elasticity of different layers .

F_a , F_b and F_c =The axial forces in different layers.

h_a , h_b and h_c = Thickness of different layers.

I_a , I_b and I_c =Second moment of area for the layer a.

I_1 and I_2 = Moment of inertia of concrete slab and steel about its own centroid.

k_{s1} and k_{s2} =Shear stiffness of the joint per unit length between successive layers.

k_{n1} and k_{n2} =Normal stiffness of the joint per unit length between successive layers.

L = span length.

M= External applied moment.

M_a , M_b and M_c =Moment for layer a.

P= Point load.

P_1 and P_2 =Normal force per unit length at the upper and lower interface.

ρ_i =Live load.

ρ =Live load and dead load.

ρ_a , ρ_b and ρ_c = Distributed self-weight of layer a.

R_r , R_l =Reaction at the right and the left supports.

U_{ab} and U_{bc} = Slip between upper and lower layers.

u_a , u_b and u_c =Displacements of the different layers in the x -direction.

W=weight of charge in Ib.

w_a , w_b and w_c =Displacements of the layer a, b and c in the z -direction.

w_{ba} , w_{cb} =Separation at the interface between the upper and lower layers.

x.= Subscript denote differentiation.

X=distance from charge in ft

Z= scale factor $= \dfrac{x}{W^{1/3}}$

z_{ai} , z_{ai} and z_{ai} =Z-coordinate of interface relative to local x-z axes in layers a, b and c.

ε_f =Free strain due to shrinkage, temperature etc.

ε_r = Strain induced during the construction sequence.

$\bar{\varepsilon}$ =Integration of strain function over cross section area of the material.

ε_a , ε_b and ε_c =Strain in layers a , b and c.

σ_a , σ_b and σ_c =Stress in layers a, b and c.

Δx =Spacing between nodes.

**Table(1) : Details of charge weight, peak over pressure and duration**

Case No.	TNT charge weight		Distance from the charge		Peak over pressure		Duration
	kg	Ib	m	ft	MPa	psi	sec
1	2.5	5.434	1.5	4.95	1.285	187.56	0.00106
2	5.0	11.2	2.5	8.25	0.576	84.0875	0.000905

Table(2) : Properties of material

	Compressive strength MPa	Modulus of elasticity GPa	Tensile strength MPa
Concrete beam	25	20	2.05
steel	Yield stress MPa	Modulus of elasticity GPa	Elongation%
Plate	290	205	24
Headed studs	285	203	22
Reinforcing bar	290	202	20
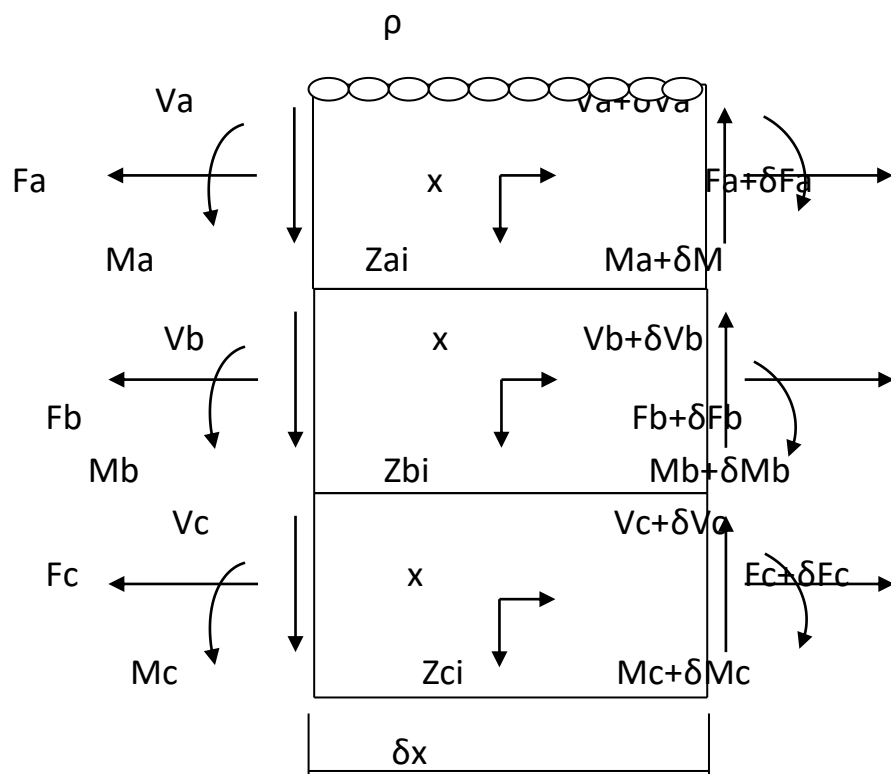
(a) Composite layered beam
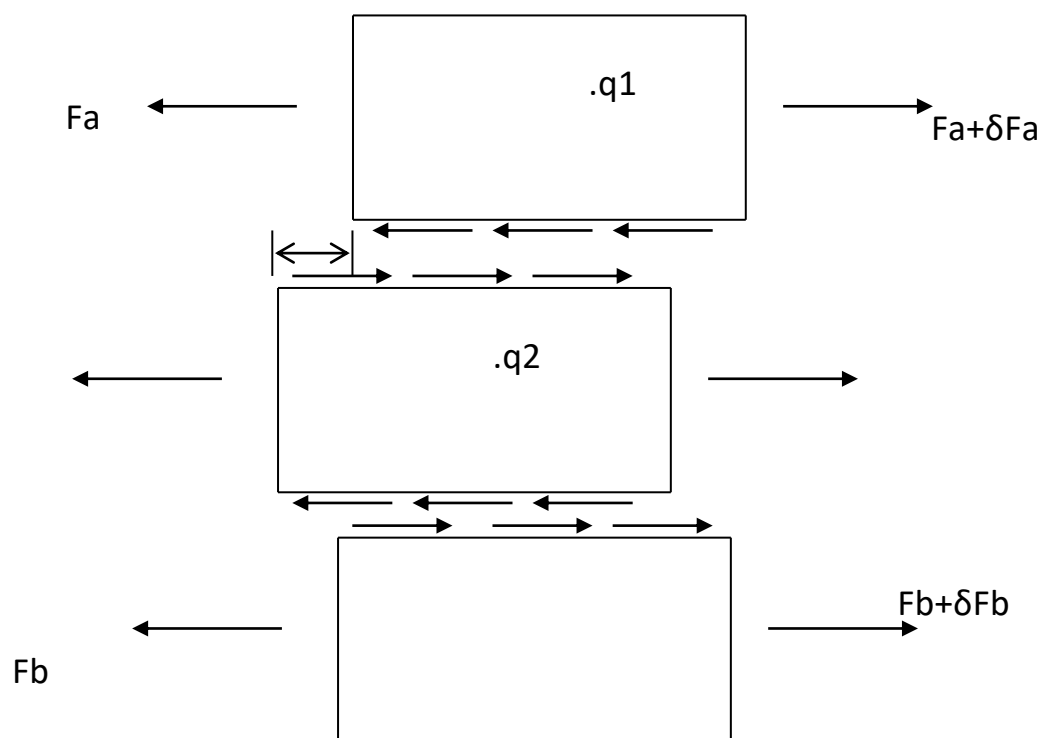
(b) Composite layers element in Slip
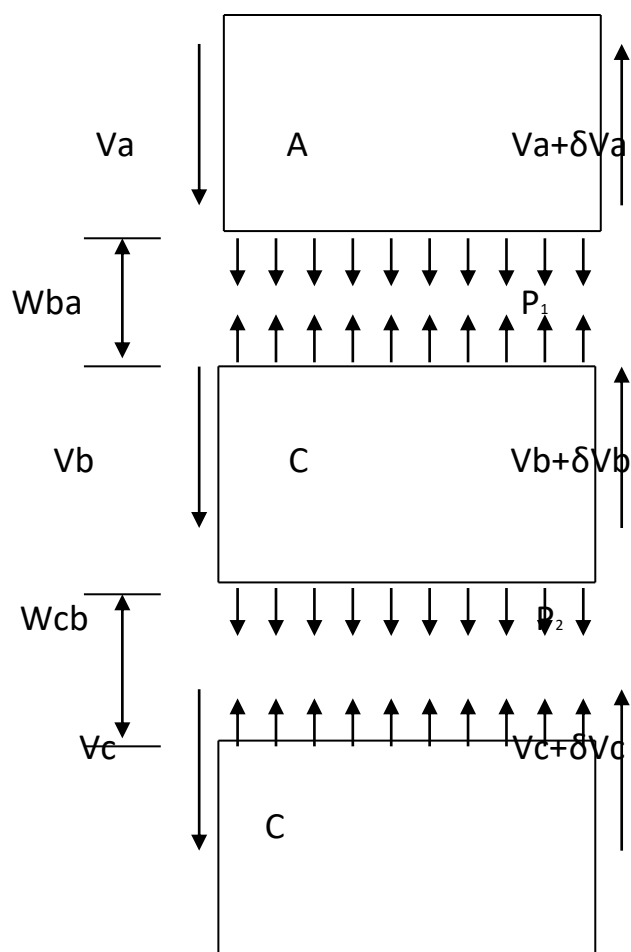
(c) Composite layers in separation

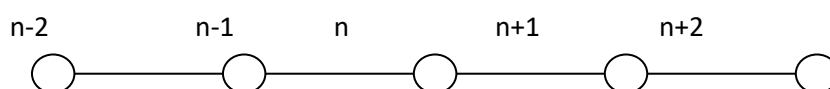Figure (2) Composite three layers element

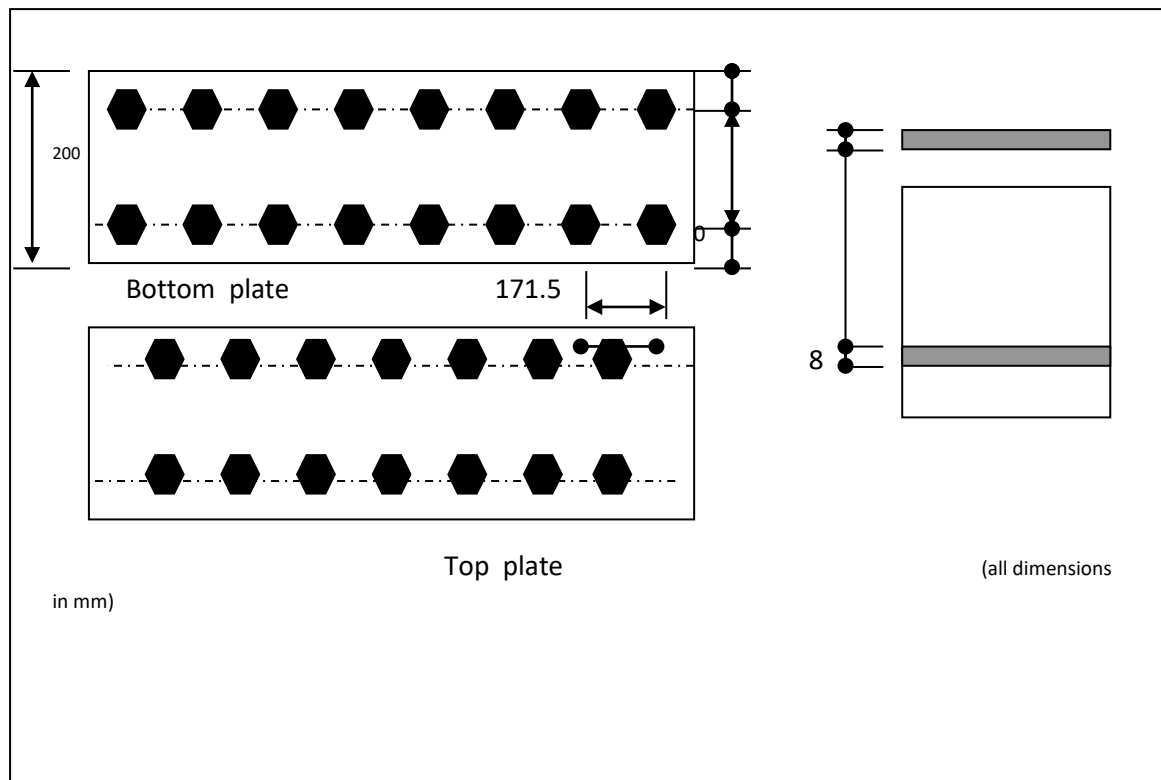
Figure (3) : Central finite difference scheme

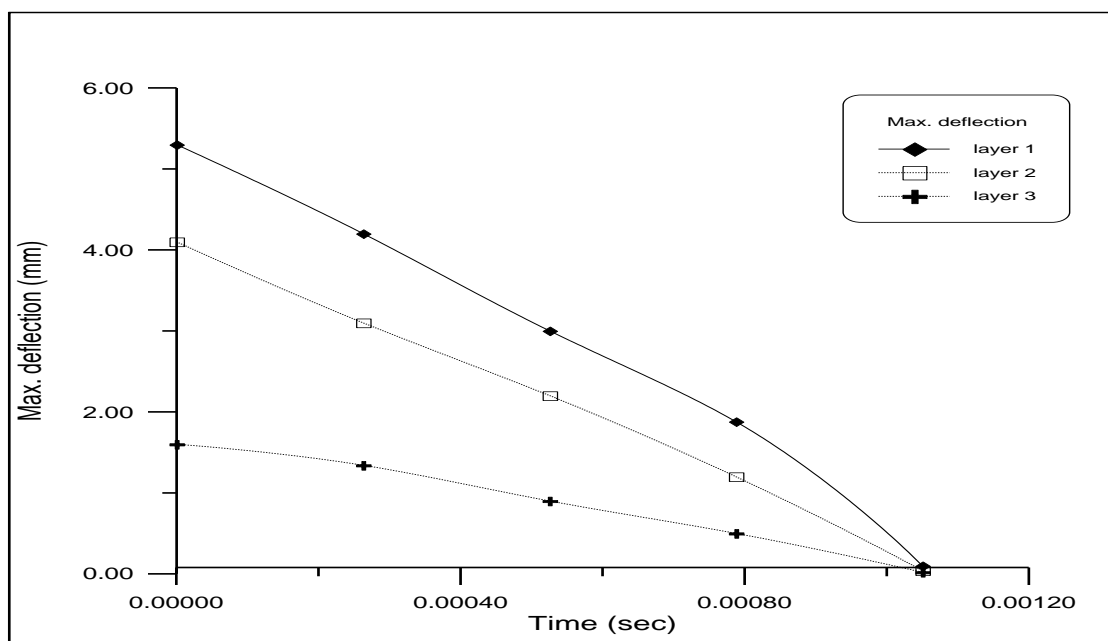Figure (4): Dimensions of beam
(Top and bottom view)

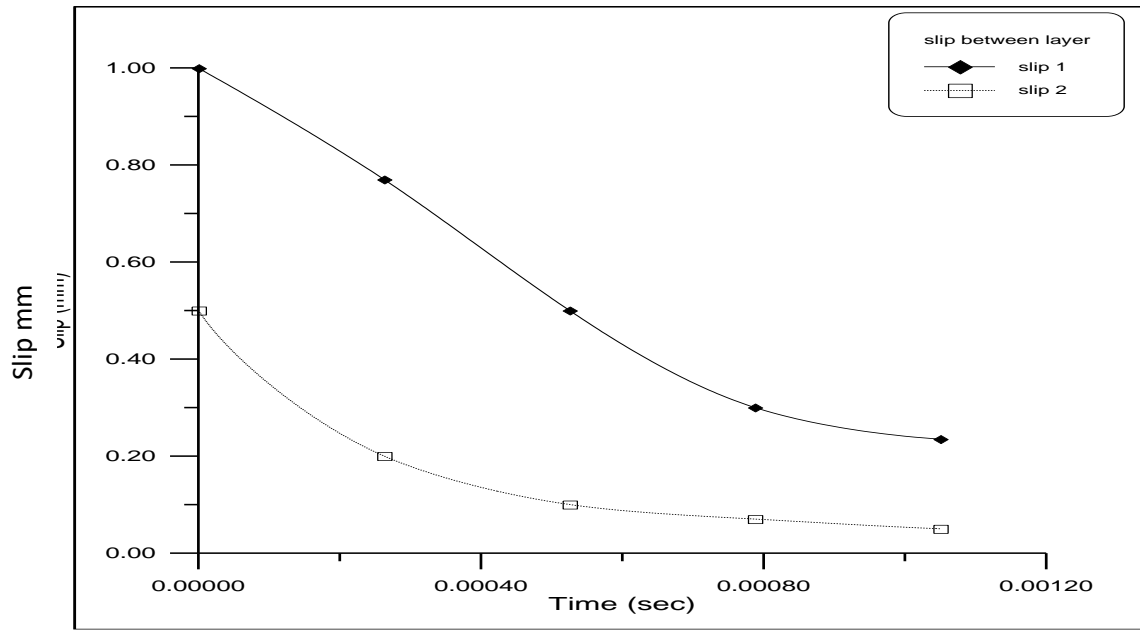Figure (5): Time-deflection relationship for beam layers under blast load

Figure (6): Time-slip relationship for beam layers under blast load

تحليل العتبات متعددة الطبقات بسيطة الإسناد تحت أحمال العصف

م.د. خليل إبراهيم عزيز   م.م. احمد طارق نعمان

جامعة الانبار – كلية الهندسة

الخلاصة

تعتبر المنشآت ذات المقاطع المركبة من المنشات المهمة نظرا" لفوائدها المتعددة, كما إن الأعضاء الإنشائية ذات المقاطع متعددة الطبقات مهمة في العديد من الاستخدامات الهندسية وخصوصا" في صناعة الطائرات والسفن. تعتبر أحمال العصف احد المؤثرات الناتجة عن الانفجارات نظرا" لقوتها والفترة الزمنية القليلة التي تؤثر بها على المنشأ . تبدأ موجة العصف على شكل زيادة كبيرة جدا"في الضغط (جبهة الموجة) ثم يهبط تدريجيا" إلى ما يعادل الضغط الجوي ويدخل مرحلة الضغط السالب (تحت الضغط الجوي). في هذا البحث تم اعتماد عتبة بسيطة الإسناد ذات ثلاث طبقات متغيرة الأبعاد والخواص الهندسية للمواد ومعرضة إلى أحمال العصف مع الأخذ بنظر الاعتبار الإزاحات الأفقية والشاقولية لكل طبقة. نتيجة التحليل تتكون ست معادلات بالاعتماد على متطلبات التوازن والتوافق تم حلها بطريقة الفروق المحددة حيث تكون الأحمال دالة للزمن . حيث وجد ان العناصر الإنشائية ذات الطبقات المتعددة هي الأكثر مقاومة لأحمال الانفجارات.