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# On the vibration and stability investigations of orthotropic FGMs plate and cylindrical shell: A review

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### ABSTRACT

Since FGM orthotropic structures have such striking qualities as high strength, exceptional stiffness, stiffness-to-weight ratio, reduced cost, and high strength-to-weight ratio, they are employed extensively in the mechanical, aerospace, and civil engineering sectors. Thick plates and shells have more noticeable shear deformation effects. Therefore, in recent years, there has been a lot of interest in the vibration and buckling investigation of FGMs orthotropic plates and shells. Moreover, researchers have developed a variety of approaches and procedures for the examination of orthotropic FGM plates and shells. The majority of the literature review in this publication is focused on orthotropic FGMs plate and shell buckling and linear and nonlinear free vibration. In engineering practices, it is customary to use material-oriented or orthotropic materials in several domains to optimize the structures and maximize material properties, which is especially crucial for FG constructions. Solutions for the orthotropic FGM structure are studied analytically and numerically with different plate and shell theories.

## 1. Introduction

Material scientists attempted to use functionally graded materials (FGMs) to provide materials that could withstand extremely high temperatures in the spacecraft project in 1984. Ever since, research on female genital mutilation has flourished in nearly every related discipline. High temperature resistance, large resistant-to-weight ratios,

anticorrosion, and anticorrosive qualities are among the benefits of FGMs, which are often composed of a combination of metallic and ceramic materials with a progressive shift in material properties from one contact to the next. As a result, FGMs are frequently utilized in thin-walled cylinder shell constructions, including pressure vessels, petrochemical plants, oil refineries, space shuttle propellant tanks, ballistic missile skins, and power

plants. The physics and mechanism of FGMs, unique promising materials for composite structure, have been thoroughly studied during the past few decades. Grounded on the materials change along the targeted directions, almost all of the investigations on FGMs can be divided into three types: power-law FGMs (P-FGMs), sigmoid-graded FGMs (S-FGMs), and exponential FGMs (E-FGMs) [1].

Modern composites are widely used in many modern technology products, which have directed the growth of original tasks and the need to consider new primary factors that determine a structure's bearing capacity in addition to the traditional approaches for examining thin-walled plates. Anisotropy and the material's heterogeneity rank highly among these variables. The analysis of the vibration and stability issues with composite constructions is made more difficult by these variables. Reddy's study [2] significantly added to the theory of anisotropic plates.

Composite cylindrical shells are widely used in the strategy of missiles and vehicles, nuclear mechanisms, and ship-building assemblies. The reliability of these structures is closely related to the functionality of their constituent parts. The dynamic performance of this structure is crucial to construct structural components for the nuclear, aerospace, and petroleum sectors. These cylinder-shaped structures often shake at enormous amplitudes, exceeding the shell's thickness. Many studies of the nonlinear vibration of shells under various shear deformation theories have been motivated by this challenge. Also, the shear stress consequence on the nonlinear response of the non-isotropic shell is more considerable than the isotropic case because of the large ratios of Young's moduli to transverse shear moduli in the composites. This phenomenon significantly drops the active flexural stiffness of nonisotropic composite plates and shell. [3]. Besides, the thin-walled cylinder shell structures have been extensively utilized in aircraft manufacturing and other manufacturing disciplines for several years because of their great stiffness, low price, and large resistant-weight ratios. Examples of its applications include the space shuttle's propellant tank, the casing of flying missiles, the air receivers tank, the distillation column, and heat exchangers-condensers. Thin walled and slender manufacturing structure must maintain stability in

addition to meeting load-carrying capacity requirements, per design standards and codes. However, dynamical loads such as winds, earthquakes, and stochastic dynamical loads are frequently and inherently functional to structure in engineering practice. Structural safety may be jeopardized if these loading conditions are oversimplified in structural analysis or design [4].

However, it is customary in engineering practices to use material-oriented or orthotropic materials in several domains to optimize the structures and maximize material properties, which is especially crucial for FG constructions. Furthermore, the FGMs are more prone to losing their isotropic and becoming nonisotropic, with the significant direction equivalent to or expected to each layers, because of the physical composition and fabrication methods used [5], [6]. For instance, categorized Cu-Ni-Sn specimens showed duplex or lamellar structures following the plasma sprig procedure, according to Kaysser and Ilschner [7]. Transmission electron micrographs (TEMs) show an equiaxed grain microstructure, a similar feature described by Sampath et al. [8]. Therefore, considering the orthotropy FG when researching the dynamical behavior of the cylinder shell is not out of the ordinary. Using various analytical and numerical techniques, this work attempts to review the literature that is currently accessible on orthotropic functionally graded materials plates and cylindrical shells that have been created in the past few decades.

## 2. Literature Survey

### 2.1 Vibration of orthotropic FGMs plates and shells

W. SOEDEL [9] solved the vibration of an orthotropic cylinder shell. The model was based on assumptions similar to the Donnell Mushtari Vlasov for isotropy shell, initial with Love-type calculations for orthotropy cylinder shell. Then, exact-form explanations for instances with simple support are produced. Finite element findings are compared with the outcomes of two sample scenarios, and the results demonstrate good agreement. It is suggested that this streamlined method simplifies evaluating the impact of adjustments to design parameters. The nonlinear vibration investigation of composite orthotropy cylinder shells was investigated by LAKIS et al. [10]. It is a cross between traditional shells theory and finite-element theory. Strain-displacement and

non-linear Sanders-Koiter relations are applied. Linearized equations of motion are used to assess displacement functions. Then, get the modal constants for these displacement functions. Further, the finite element technique was applied to determine the linear, nonlinear stiffness, and mass matrix expressions.

Ganesan and Sivadas [11] investigated the vibration response of the orthotropy cylinder shells grounded on Love's shell model and using the finite element solution with two types of clamped and simply-supported edge conditions and the thickness varying along the axial direction. Additionally, the impact of thickness distribution and the orthotropy parameters on natural frequencies were studied thoroughly.

To forecast the properties of the modal for the thin walled cylinder shell with the freely end, IP et al. [12] created an analytical model. The shell possesses mid-plane symmetry and is orthotropic. A strain helpful energy for both twisting and elongating effects is developed by Love's first calculation shell theory. Next, the shell vibration mode forms are simulated using typical beam function with the Rayleigh Ritz variational technique. Test results validate the model's correctness. Using the established model, it is possible to identify Rayleigh and Love modes with near frequencies in extensional modes. The contributions of different elastic characteristics to the strain energy are also examined. The findings indicate that the vibrating structure's circumferential modulus contributes significantly to its flexural energy, whereas its longitudinal and inplane shear modulus mostly subsidizes its stretching energy. Furthermore, correspondingly, a significant rise in the ratios of energies related to the longitudinal and shear modulus was found with a reduction of the shell thickness. With minimal changes to natural frequencies, it is possible to increase the bending and twisting resistance of shells by altering the lamination stacking order.

A government equation with flexible factors is constructed in a combined matrix formula from the three-dimensional essential calculations of anisotropy elasticity via Chen et al. [13]. Next, we study the vibration of fluid-filled, simply-supported, cylindrical orthotropy, FG cylindrically shell with random thickness. An approximation laminate model is used to account for a random fluctuation of materials coefficients along the

circular axis. A comparison is made between the numerical examples and the current findings. Finally, the impact of associated characteristics on natural frequencies is explored.

For both forced and free-vibrating of FG cylindrically shells, a precise elasticity solution was given via Vel [14]. The conditions of the FG shell were simply supported, and the radial materials gradation was chosen to be arbitrary. Essential to the state of general plane-strain deformations in the axial axis, the three dimension linear elastodynamics equation was explained with appropriate displacements function that fulfill the edge conditions identically. However, the power series approach was used to analyze the subsequent systems of coupled ordinary differential equations with inconstant factors analytically. The analytical solution can be applied to deep and shallow shells with any thickness. Although the formulation was equally valid to the exceptional situation of isotropic materials, it presupposed that the shells were modeled of orthotropy material. Moreover, the results for fiber-reinforced and two-constituent isotropic composite materials were given. The normalized elastic stiffness of isotropy material was assessed using the self-consistent technique. The asymptotical growth homogenization or Mori-Tanaka approaches were used for the reinforced fiber material to determine the effective characteristics. The silicon-carbide fibers used in the fiber-reinforced composite material under study were inserted in a titanium matrix, with the orientation of the fiber and the volume percent graded radially. Finally, the study gave varied results regarding the natural frequencies for various factors such as mode forms, displacements, and stresses for various shell geometries and material gradations.

Zhang et al. [15] examined the non-linear dynamics and chaos of flat plates composed of supported orthotropy (FGMs) in a temperature environment when exposed to external and parametric excitations. Two factors were considered: temperature-dependent material characteristics and heat conduction. According to a straightforward power-law function with respect to the constituents' volume fraction, the material attributes were ranked in the thickness direction. Hamilton's concept was applied to generate the main motion calculations for the orthotropic FGM rectangular plate based on Reddy's third order shear stress plates theory. A nonlinear formula

with three-degrees of freedom was obtained by using the Galerkin process to the partial differential main motion equation. However, internal resonance with main parametrical resonances and subharmonic resonances with a power of  $1/2$  was the resonant case that was being examined here. The orthotropy FGMs flat plate's periodic and chaotic motions were examined using the phase representation, wave-form, and Poincare map, all based on the averaged equation produced using the many scales approach. Under some circumstances, it was discovered that the orthotropic FGM plate's motions were chaotic.

Sofiyev AH et al. [16] offered an analytically model to examine the dynamically response of a FGMs orthotropy cylinder shell with elastic substance and under mutual effect of the axial compressive, tension loads and ring-shaped compressive pressures with fixed velocities on the grounds of the vibrations theory of shells. To illustrate the effects of different factors on the dynamical displacement, dynamical influences, and the critical value of velocities for shell, a parametric study was carried out. These parameters include Winkler or Pasternak foundation, the nonhomogeneity and orthotropic factors, the thickness ratios, and the velocities of the loads.

Liu B et al. [17] provided an analytical process and exact form for the vibration explanations of orthotropic cylindrical shells with classical edge conditions grounded on Donnell's shells theory. In this study, the closed-form natural frequencies are successfully produced by using the method of separation variables in the formula of a small package. Additionally, the properties of the eigenvalues were investigated. The semi-analytically differential quadrature finite element method technique computed by the presented approach and comparisons with solutions found in literature were used to validate the exact solutions.

Sofiyev and Kurguoglu [18] introduced and analytically solution for vibration and instability of functionally graded materials (FGMs) orthotropy cylinder shell subjected to pressure and using first order shear deformation theory (FSDT). The governing equation of motion for the shear deformable FGM orthotropic shells was obtained grounded on the Donnell's theory and explained using the Galerkin's techniques. Moreover, the impact of shell features, orthotropic, compositional

behavior, shear deformation on the nondimensional frequencies factor and the critical outside pressure were studied parametrically. In addition, to illustrate the distinctions between the shear deformation theory of shell and several higher order shell shear deformation theory, some comparisons between different theories have been done.

Najafov et al. [19] considered the vibration of FGMs orthotropic cylinder shell, based on the shear deformation theory with the assumption of that the Young, shear moduli and the density of the orthotropic materials were continuous change in the thickness path. Constructed on the Donnell type theory, the motion equations with the effect of rotary inertia were obtained. Through application of the Galerkin method, the fundamental equations were decrease to the sixth order algebraic equations for the frequency; answering these algebraic equations yields the lowest values of dimensionless frequencies parameter for nonhomogeneous orthotropy cylindrical shell with and without rotary inertia and shear deformation. Further, the impacts of shell geometry parameters, orthotropy, non-homogeneity, rotational inertia, and shear stress on the lowest value of the nondimensional frequency factors were discussed along with the calculations. The acquired values were compared with those found in the published literatures to validate the proposed method.

Sofiyev and Kurguoglu [20] provided an analytical solution for the nonlinear dynamical response of FG orthotropy cylinder shells resting on elastic foundations using shear deformation theory combined with the von-Karman relationships. Material factors of the shell change ongoing based on the exponential type within the thickness coordinate. Moreover, the homotopy-perturbation scheme (HPM) was operated to resolve the nonlinear ordinary differential equation, obtained by applying Galerkin and superposition procedures. The validity of the presented approach was carried out by contrasting the outcomes with those found in the literature for the limited instances. The impacts of the non-linear elastic foundation, vibrating amplitudes, shear stress, heterogeneity, and shell parameters on the non-linear frequency factors were presented.

A finite element (FE) approach was introduced by Baccocchi and Tarantino [21] to investigate the

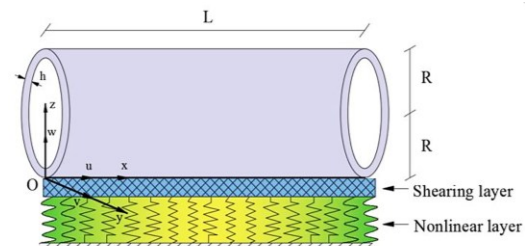
natural frequencies of orthotropic laminated plates that are functionally graded and have cross-ply layers. For this, a nine node Lagrange elements were taken into consideration. The study's primary innovation was the modelling of the orthotropic layers' reinforcing fibres under the assumption of a nonuniform distribution in the thickness coordinate. The total properties of the composite layers were defined using the Halpin-Tsai technique, which begins with the characteristics of the two elements. A number of functions are presented to characterize the dependence of their volume fractions on the  $z$  axis. The shear deformation shells theory (FSDST) for the thick plate provided the theoretical framework for the studies. However, the shear stiffness of the structure is disregarded while handling the vibration analysis of thin plates using the same methodology. This goal is accomplished by accurately determining the shear correction factor value without changing the formulation in any way. The findings demonstrate that the non-uniform fiber arrangement along the thickness direction has an impact on the essential frequencies and the vibrating mode of the dynamic response of both thick and thin plates.

Sofiyev [22] discussed the high amplitudes vibration investigation of FGMs orthotropy cylinder shells within non-linear Winkler elastic foundations within Donnell's shells theory and the von Karman 's-type geometric non-linearity relations. The motion equations were changed into a nonlinearity ordinary differential equations form using the superposition and Galerkin approaches. The semi-inverse style was applied to achieve the frequency-amplitude behavior of the FGMs orthotropy shell. By contrasting the outcomes with the published results found in the literature, its accuracy was confirmed. Furthermore, some novel conclusions for the nonlinear frequencies parameters of the cylinder shells were also presented to examine the impacts of the non-linear elastic foundations, vibrating amplitudes, FGMs orthotropy factors, and shells features.

A mathematical framework for the vibration characterization of a thin, orthotropy, generally FG flat plate with a central internal fracture was studied by Joshi et al. [23]. The uninterrupted line fracture runs parallel to one of the plate's borders. The equilibrium concept is used to determine the orthotropic plate's equation of motion. The Line-Spring Model was applied to create the crack

words. The Line Spring Model's crack obedience constants were introduced to analyze how the placement of the crack through the plate's thickness affects natural frequencies. The cracked plate equation of motion is obtained and converted to a cubic non-linear system via Berger's modal for the inplane forces. Additionally, the well-known Duffing formula was found by using Galerkin's approach to the equation. Using the Multiple Scales perturbation approach, the peak amplitudes was achieved. The influence of nonlinearity is also established by utilizing the many scales approach to get the frequency-response form for the broken plates. It was proven how the length of the fracture, the edge conditions, and the placement of the break along the thickness impacted the frequencies of a flat plate. It is discovered that the length and placement of the fracture along the plate's thickness have an impact on the vibration characteristics.

Sofiyev et al. [24] accomplished analytically results for the nonlinear vibration behavior of orthotropy cylindrical shell with nonlinear elastic foundation grounded on the shear deformation shells theory and von-Karman's form of geometric non-linearity. Moreover, to transform the main equations into a non-linear ordinary differential equations, the superposition and Galerkin procedures were applied. The Jacobi elliptic form was applied to get the frequency-amplitude behaviour for orthotropic shell within the context of the SDT. Furthermore, associating the achieved result with those found in published literature demonstrated the validity of the suggested approach. additional, nearly new findings for the non-linear frequency aspects of the cylinder were presented in order to explore the influences of vibration amplitudes, shear stress, orthotropy constraints, and the elastic foundation on the shells properties. Figure 1 shows the geometry of the FGMs orthotropy cylinder shells enclosed by a nonlinear elastic foundations.



**Figure 1.** Orthotropic FGM cylindrical shell geometry sitting on a non-linear elastic base.

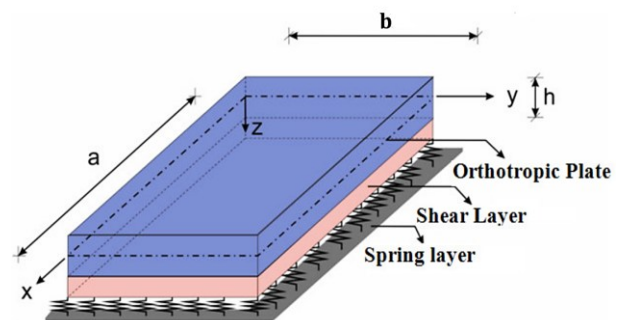
The vibrating response of the FGs orthotropic cylinder shell with elastic foundations was investigated analytically by Sofiyev et al. [25]. The first-order shear deformation shell theory with the von-Kármán's-type of strain displacement relations were applied to accomplish the dynamical stability and compatibility equation of the FGs orthotropic cylindrical shells. Furthermore, the motion equations were converted into non-linear ordinary differential equations by applying Galerkin and superposition processes. Founded on the first order shear deformations shell theory assumptions, the homotopy-perturbation scheme (HPM) expresses the nonlinear frequency of the orthotropy cylinder shell of the FGs. More precisely, a similar conclusion was also generated within the assumptions of the traditional thin shells theory. Additionally, the outcomes were confirmed and contrasted with those reported in the literature. The investigation's goals were finally reached by calculating and visualizing the impacts of various parameters that were part of the examination.

Sofiyev [26] introduced a nonlinear free vibration of FGs orthotropy cylinder shell with considering the shear stress effect and grounded on first-order shear deformation shells theory and the von-Karman's nonlinear strains-displacements relationships. The FG orthotropic cylindrical shells' equations of motion were obtained, and the main equation for the motion was then transformed into a nonlinear ordinary differential equations by using the superpositions and Galerkin's approaches. The Jacobian elliptic scheme was introduced to achieved the equations for the nonlinear frequencies and the nonlinear to linear frequencies ratio that depend on the amplitude based on the FSDT. The effects of nonlinearity, shear stress, FGM parameters as well as the shells features on the nonlinear frequencies were examined through a comprehensive parametric result.

A microstructure-dependent model was presented via Yang and He D [27] for the free vibrating and instability examination of an orthotropy FGs microplate in light of a re-modified couple stresses theory. Two material length scale parameters were introduced to account for the macroscopic and microscopic anisotropy simultaneously. However, it was believed that the attributes of the material would fluctuate ongoing throughout the thickness axis regarding a power-law model. Then, the main

equations and matching boundary conditions were determined with Hamilton's principle. The normal frequency and stability loads of a simply supported microplates were computed using the Navier method. According to the numerical outcomes, the current technique outpredicts larger normal frequency and critical stability load than the classic technique, especially when the micro-plates' geometric size is similar to the parameters of the material length scale, indicating a well-represented scale effect. As the power law index or anisotropy diminish or the material extent scale factors rised, the scale impact becomes more apparent, and vice versa.

the vibration behaviour of the exponentially FGs orthotropic plates resting on the elastic bases was examined by Hacıyev et al. [28]. The heterogeneity of the orthotropy changed exponentially in the thickness and axial directions. Moreover, the governing equations from the traditional thin plate theory were solved using the Galerkin scheme. Lastly, a comparison with past studies was conducted to confirm the results that were reported. The influence of the orthotropy parameters, material gradient factors, and parameter elastic foundation on the dimensional frequency factors (DFFs) was investigated. Figure 2 displays the geometry and coordinate system of the orthotropy FGs plate.



**Figure 2.** Dimensions and geometry of a FGs orthotropic plate supported by an elastic Winkler-Pasternak foundations.

A straightforward analytical model was used by Ghashochi-Bargh and Razavi [29] to simulate the vibration of orthotropic, FGs flat plate. By means of Mindlin's plates theory, the main equations of the transverse displacement and rotation for the plate's mid plane have been determined. The rotation parameters have been replaced through

the transverse equation of the plates after being represented in terms of transverse displacement. Next, for each edge condition, a suitable form for the transverse displacement of the plates has been assumed. Finally, the normal frequencies for the plate have been definite by using the orthogonality of these functions. A few instances have been used to validate the proposed paradigm.

Fu et al. [30] suggested a modified Fourier-series form for the free vibrating of orthotropy FGMs thick plate under general boundary restrictions in light of the first-order plate theory. To avoid all border discontinuities and jumps, displacement and rotation of each plate were defined as an enhanced variant of the double Fourier cosine series plus many closed forms supplementary functions, regardless of boundary limitations. Grounded on Rayleigh's-Ritz's approach, the energy function for the plates yields exact answers. The impacts of geometric parameters, volume fraction, and gradient index on frequencies with broad boundary constraints are shown.

A Chebyshev spectral technique was introduced by Huang et al. [31] to analyses orthotropic in-plane FGMs (IP-FGMs) plates for bending and free vibration under generic boundary conditions. Material qualities and distributed loadings are believed to change arbitrarily in the plate plane. Gauss-Lobatto sampling combined with high-order Chebyshev expansions approximates both of them. Then, using the Lagrange equation and the principles of the overall potential energies, the motion equation for the vibration and bending were accomplished for the orthotropy IP-FGMs plate, respectively. In light of the Mindline-Reissner first-order plate theory. These equations were reliable for many type of materials and loading distributions. A numeral of mathematical examples were provided to reveal the precision and effectiveness of the suggested procedure, involving different boundary conditions, loading kinds, and thickness-to-length ratios.

Jain et al. [32] presented an analytical explanation for the vibration examination of an orthotropy and FGMs underwater cylinder shells with a surface fracture of flexible angular direction. Classical thin shell theory had been used to construct the main equations with the transverse deformation of cracked submerged shells. Furthermore, the fluid-structure contact consequence was incorporated into the governing differential equation by adding

the fluid reactions related to its inertial impacts. To account for the impact of crack in the governing equation, the crack coefficients were expressed via the line-spring formula (LSF). Moreover, the fundamental frequency was obtained by solving the governing equation with the aid of Donell's Mushtari-Vlasov theory. Finally, the findings for frequencies were displayed using the input parameters for instance; the radius, thickness and length of the shells along with the crack orientation and length.

Sourki et al. [33] analytically obtained thick laminates' vibrating absorption with rectangular FGMs orthotropic cross-ply. The von Kármán strains relations containing highorder shear theory were used to analyze the plate. The laminate structure was expected to perform as intended in a temperature environment with in-plane loadings. Additionally, all the layers in the plate comprise an orthotropy composites lamina, and mechanical properties were supposed to change continuously with thickness. The main equations were generated via Hamilton's principles and the Levy-type solutions to implement several boundary conditions, such as simply-support, free, and clamped on the structure's edges. Taking into account the in-plane thermal stresses, a specific solution was obtained. The impact of temperature fluctuations, in-plane loads, and edge conditions on the plate's natural frequency was measured. Ultimately, the in-plane forces have been created with a magnitude that can effectively suppress plate vibration under all thermal and barrier conditions.

In their study, Ali and Hasan [34] examined the vibration behaviour of large-amplitude FGMs orthotropy toroidal shells, including convex and concave shell segments (FGMOTSS). The characteristics of the material were supposed to change in z directions based on exponential-law. The nonlinear equations were founded on the von-Karman's kind of non-linearity and the hypothesis of Stein and McElman, as well as shear deformation theory. The nonlinear ordinary differential was found by using the Galerkin scheme and superposition technique to the non-linear partial differential equations of FGMOTSS. The formula for frequency-amplitude relations and the nonlinear frequencies to linear-frequencies ratio of FGMOTSS were extracted by solving nonlinear equations using the harmonic equilibrium method. Basic analyses are performed using the homogeneous

orthotropy and orthotropy FGMs for the convex and concave shells. Their large-amplitude vibrational behaviours are compared, and many instances demonstrate the influence of heterogeneity.

## 2.2 Buckling of orthotropic FGMs plates and shells

Pelletier and Vel [35] examined the functionally graded thick orthotropic cylindrical shell's steady-state behavior when it was exposed to mechanical and thermal loads. It was assumed that the functionally graded shell had a random variation of material qualities lengthways its circular axis and was supported at its edges. Analytical solutions were established for the three-dimensional steady-state heat conduction and thermo elasticity expressions, which were reduced to global plane-strain deformation in the axial axis. The thermo-elastic governing equations were condensed to a scheme of coupled ordinary differential equations with inconstant factors using appropriate temperatures and displacement functions that, In the same way, fulfill the edge conditions at simply supported edges. Consequently, these equations were then explained employing the power-series scheme. Although isotropic materials can also use the analytical solution, the cylinder shells in the current formulation was assumed to be composed of an orthotropic FG material. Moreover, there was a smooth fluctuation in material volume fraction and/or inplane fiber alignments along the radial coordinate for two essential isotropy and reinforced fiber FGMs shells. Additionally, the concepts of Flügge and Donnell shells were also used to the analysis of the cylindrical shells. Finally, the effects of angular span, shell thickness, and transverse shear stresses were identified by comparing the displacement and stress from the shells theories through the three dimension precise result.

Wang and Sudak [36] provided, in a general way, the examination of a thermo-elastic multi-layered cylinder panel consisting of an oblique pile of FGMs layers with orthotropy material attributes. The panel was at zero temperature and its two edges are supported. Applying steady-state mechanical and thermal boundary conditions to the cylindrical panel's outer and inner surfaces was possible. Additionally, it was supposed that the functionally graded material has a radial power law type, and the consistent solutions in piece layer were found

using the pseudo's Stroh's formalism. In addition, considering the transfer-matrix technique, the multi-layered cylindrical solution was obtained. Further, the paper presents a new, more succinct formulation of the transfer matrix approach that was yet similar. The obtained solutions were then used to examine a cylindrical panel with five layers. The impacts of material classification and ply-angles on temperature, displacement, and stress distribution were examined. Finally, the findings unequivocally demonstrated that the ply angle and material gradation significantly impact the temperature and elastic fields.

Asemi and Shariyat [37] studied the orthotropic FGM plates' buckling has. The work develops a very accurate finite element elasticity formulation based on nonlinear three-dimensional energy to investigate buckling in anisotropic FG plates with arbitrary orthotropic directions. Transverse heterogeneity and in-plane orthotropy were assumed for the material properties. The formulation and results included uniaxial compression, compression, and tension to compression biaxial loading circumstances. The notion of minimal total potential energy was the foundation for developing the governing equations, which were then solved using finite-element orthogonal integral equation. To attain the highest level of accuracy, a complete compatible Hermitian's elements with 168 degrees of freedom were utilized, providing continuousness of the strains stress component at the mutual boundaries and the element nodes from the outset. To get useful conclusions, a range of loading groupings, orthotropic direction, and aspect ratio were considered and explained in the results section. Moreover, according to the results, higher strengths may be attained when external loads are applied in the materials' major directions.

Using the Galerkin approach, Najafov et al. [38] examined the torsion vibration and instability issues of (FGMs) orthotropy cylinder shells with elastic intermediate. The response of the elastic intermediate on the cylinder shelle was explained by the Pasternak model. Furthermore, the mixed boundary conditions were taken into account. It was expected that the density and material characteristics of the orthotropic cylindrical shell change exponentially with thickness. The fundamental formulas for the FGMs orthotropy cylinder shells under torsional loads originated on

an elastic intermediate of the Pasternak type were obtained. For the FGMs orthotropy cylinder shells sitting on elastic intermediates, the equations for the critical torsional loads and nondimensional torsional frequencies parameters were found. Additionally, variations in the shear subgrade moduli of the foundation, orthotropy, foundation stiffness, and exponential factor indicating the degree of material gradient all impact the nondimensional torsional frequencies parameter and critical torsional load.

Shear buckling analysis of orthotropy heterogeneous FGMs plate was examined for the first time by Shariyat and Asemi [39]. Additionally, the impact of the Winkler-style elastic base was taken into account. Transverse heterogeneity and in-plane orthotropy were assumed for the material properties. Rather than applying the approximative plate theories, the most precise method, three-dimensional elasticity, was used. Using the suggested 3D cubic B-spline element, present formulations were C2-continuous, in contrast to all previous displacement-grounded buckling assessments using C0-continuous commercial finite element models or semianalytical approaches. The lowest potential energy principle and a nonlinear finite element method using a Galerkin-type 3D cubic B-spline key algorithm were used to get the results. Besides, a generalized geometric stiffness concept was used to detect buckling loads. Effects of the buckling and prebuckling states were considered in this regard. Although, for the plate with supported edge, specifics of the stability mode form and the foundations interaction were examined to provide a clearer picture and more in-depth discussions. Furthermore, consideration was given to the more realistic free and clamped edge circumstances.

Sofiyev et al. [40] provided an analytical formulation for the stability analysis of orthotropy (FGMs) cylinders with shear deformation effect and under lateral pressure, which is the foundation of shear deformation theory and the classical thin shell theory. In addition, parametric studies were conducted to find out how shear deformation, orthotropy parameters, and heterogeneity affect the dimensionless critical pressure.

Asnafi and Abedi [41] examined the bifurcation incidence and dynamic buckling under lateral stochastic pressures for three well-known plate types: orthotropy sigmoid, power function, and

exponentially FG plates. The behavior and analysis were not typical deterministic research because of the randomness. Therefore, the probability density function of the response was used to estimate the dynamic stability zone and bifurcation border curves. The latter was calculated using a Fokker Planck Kolmogorov equation that had been solved exactly. The bifurcation boundary surfaces and the three-dimensional dynamic stable zone were determined based on the material parameters, in-plane loads, and the mean lateral loads. In order to make the conclusions more widely applicable, all of the parameters were converted to appropriate, nondimensional factors, and the impacts of each mandated parameter on dynamic stability were thoroughly examined and contrasted. Furthermore, the plates were compared to each other and to the matching homogeneous plate. Lastly, the bifurcation diagrams of the nondimensional deflections of the plate, which were derived straight and quantitatively from the plate main equations, validate the results.

The thermo-mechanical buckling was investigated in hygrothermal conditions via Mansouri and Shariyat [42] using orthotropic auxetic plates with negative Poisson ratios. When the auxetic plates were made of FG orthotropic material and had an elastic basis surrounding them, the complexity rose. For the first time, the analyses above were performed in this study. Biaxial or uniaxial buckling loads were both possible. Additionally, material qualities that were reliant on moisture and temperatures were occupied into the explanation. The article also considered the pre-buckling effects into account. However, an original differential-quadrature technique (DQM) was used to solve the high-order shear deformations' main differential equations. The final solution might cover numerous useful, easier applications. Furthermore, a varied series of geometric and material features parameters and different boundary circumstances were successfully studied parametrically. Additionally, the results revealed that the buckling strength and material properties were degraded by hygrothermal conditions, particularly for higher gradation exponents. Elastic foundations can improve buckling behavior by observing the buckling pattern. As the orthotropy angle increased, the buckling load decreased. Finally, authenticity has been found to reduce buckling strength for the material and environmental data used.

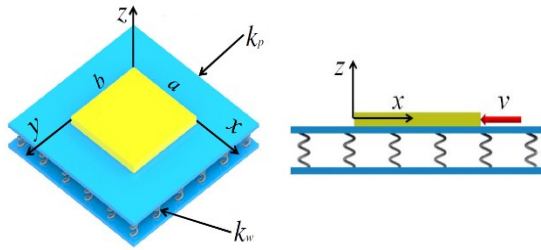
Abedi and Asnafi [43] presented an analytical determination of the stability district and boundary curve of bifurcation for a power law functionally graded orthotropy plates excited by lateral white noise. The orthotropy power law FG attributes were used to compute and illustrate the instability region as a load function. First, the general power function FG plates governing equation was derived about an assumed material attribute. Subsequently, it was reformulated with the addition of a few nondimensional constraints, making the outcomes useful and appropriate to a large variety of plates. Then, a precise relative for the response probability density function was constructed using the Fokker-Planck-Kolmogorov equation. Moreover, to create a three dimension instability area which boundaries indicate the occurrence of bifurcation, a root locus research was conducted on the roots of the probability density functions. The method's accuracy to anticipate the instability and bifurcation was verified by an example, without sacrificing generality. Additionally, when some analogous figures were created to compare the behavior of homogenous plates with their matching functionally graded ones, the effect of the material property was explored. Lastly, the results were verified by the numerical bifurcation analysis of the plate and the Monte Carlo simulation.

Abedi and Asnafi [44] examined every parameter that influences the non-linear behavior of S-FGM plate when subjected to stochastic white noise excitations along the lateral direction. First, the governing equation for the hypothetical problem was derived for a general S-FGM plate. After that, it was rebuilt with the addition of a few nondimensional parameters, making the outcomes relevant to a large variety of plates. The actual characteristics of the buckling and bifurcation of the transverse vibrating of the plate were explored, counting the material property and the mean values of the sideways loads with the inplane loads, without losing generality and using an example. Particular emphasis was placed on the importance of material properties, and several analogous figures were created to contrast the behavior of the FGMs plate with that of the homogeneous ones. Lastly, it was demonstrated that material properties can influence the behavior of stability and the occurrence of plate bifurcation.

Most of the literature currently accessible on plate buckling analysis has focused on well-behaved

designs, such as rectangular or circular geometries. Thermal buckling of generic quadrilateral plates made of auxetic (negative Poisson ratios), heterogeneous, orthotropic materials resting on Winkler Pasternak type of elastic media was studied by Mansouri and Shariyat [45]. The plate's edges can be clamped or just supported. From the perspectives of materials, edge conditions, and, to some extent, geometries, the problem was thus fairly generic and may span a wide range of actual application as a distinct case. However, the instability equations were obtained by discretizing the resultant equations by means of the differential quadratic technique after the motion equations of the plates were transformed from geometrical rectangular cartesian axis to computational natural coordinate. Pre-buckling and buckling onset scenarios have been examined to complete buckling analysis. The impacts of the foundation stiffnesses, auxeticity of the materials, heterogeneity factor, orthotropy angle, edge conditions, and skew angle of the overall quadrilateral plates on the buckling temperatures increase were finally thoroughly examined.

An analytical scheme for non-linear stability characteristics of an eccentrically composite orthotropic plate with two type of elastic foundations and under different axial velocities was offered by Gao et al. [46], with the combination of the damping influences and in the thermal effect. The non-linear compatibility equation was obtained by applying the Von-Kármán non-linearity relations with the thin plates theory. The non-linear stability equation which considered the thermal and damping influences was introduced by using the Galerkin process and stress function. Subsequent, the fourth order-Runge Kutta system was utilized to solve the obtained non-linear differential equation numerically. The Budiansky-Roth criterion as employed to evaluate the non-linear dynamic stability loads. Furthermore, the non-linear dynamic stability of the orthotropic plates was examined in relation to numerous impacts of velocities, damping ratios, temperature changes, stability modes, initial defects, and foundations parameters. Figure 3 illustrates the dimensional and geometric of a composites orthotropy plates positioned on a Winkler Pasternak elastic foundations exposed to a constant axial force.



**Figure 3.** The dimensional and geometric of a composites orthotropy plates positioned on a Winkler Pasternak elastic foundations exposed to a constant axial force.

Unidirectionally, fiber-reinforced composite with changeable fiber volume fractions had dissimilar orthotropy stiffness factor fluctuations, a defining trait that has not received much attention in the literature. With particular reference to supported rectangular plate, the goal of Bhaskar and Ravindran [47] work was to consider them and derive a three-dimensional elasticity solution. A power series approach was used in the analysis to solve governing equations with variable coefficients. A specific power function fluctuation of the volume fractions was reported for graphite/epoxy plates with a sandwich-similar structure to provide helpful benchmarks for future comparisons. The results demonstrate that displacement and stress fluctuations with thickness were very nonlinear and that the traditional plate theory did not adequately account for them. Furthermore, based on the elasticity solution, it was demonstrated that the advantage of utilizing a sandwich-like design compared to a homogenous plate varies with the span-to-thickness ratio and diminishes noticeably with increasing plate thickness.

Through the use of shear deformation theory and the damping impact, Ali and Hasan [48] examined the non-linear dynamic stability of the exponentially-law graded orthotropy toroidal shells segment with constant loading rate. According to the exponential distribution function in the shell thickness coordinate, the qualities of the shell material are expected to be graded. To produce the theoretical formulations, the shear deformation shells theory with von Karman's of nonlinearity, Stein-McElman assumptions, initial imperfections, and damping effects were used. In light of the Galerkin's approach and the fourth order Runge Kutta performance, the nonlinear dynamic stability equation is solved. Using the Budiansky-Roth criteria, the dynamical buckling

load was assessed. Furthermore, the properties of several factors on the dynamic buckling were thoroughly considered. These include geometrical parameters, velocity, defects, damping ratios, and nonhomogeneous characteristics.

Bacciocchi [49] attempted to examine the impact of the non uniform distributing of the adapted fiber through the thickness axis proceeding to the critical buckling loads of three-phase FGMs orthotropic plates made of CNT, polymer, and fiber. Carbon nanotubes (CNTs) elements and conventionally focused on the straight fiber were used to strengthen the different plies of the laminated plates. The (FGMs) reinforcing fibers along the thickness coordinate produced such layers their orthotropic characteristics. However, supposed to be aligned and changed in the thickness axis, CNTs were often the only reinforcing phase in the literature. Instead, the matrix's characteristics were further enhanced by associated, changed, straight, and oriented fiber, which were evenly spread and randomly orientated CNTs. Rather of using the typical patterns seen in the literature (like FGMs-X and FGMs-O), which were be incorporated as specific examples in the suggested technique, an overall power function was provided to describe the nonuniform characteristics. By contrasting the existing methodology with the results found in the literature, it was put to the test. The process of validation was conducted for two-phase composites, taking into account carbon nanotubes as straight, associated reinforcing fiber with uniform and changed characteristics. Additionally, a number of boundary conditions were examined and confirmed. Lastly, the paper's computational results demonstrate that variations in the fibers volume fraction's through the thickness distribution can significantly alter the values of the uniaxial and biaxial critical stability load of thin and thick plate that were arbitrarily restricted. Both the mechanical analysis and the manufacturing process of these structures should consider this influence.

Singh et al. [50] presented a precise three dimensional model for the free vibration analysis of in-plane FGMs (IPFGMs) orthotropy rectangular plate combined with piezoelectric sensing layer in terms of elastic and viscoelastic properties. Layer-by-layer unidirectional linear functional gradation in the orthotropic composite layers' density and stiffness can be considered by the established analytical procedure. Power series-grounded extended Kantorovich technique (EKT) in

conjunction with Fourier series was used to develop 3D piezo-elasticity-grounded governing motion equations in mixed formula and solve them analytically for Levy-type support conditions. To guarantee the point-wise interlayers' continuousness and electromechanical support conditions, the displacement, stress, and electrical factors (electrical field and electrical potential) were explained as the main factors. The Biot model was utilized to define the visco-elastic properties of the orthotropy interlayers. In addition, this model has similarities to the typical linear visco-elastic form. The effectiveness and accuracy of the current mathematical form were determined by contrasting the numerical outcomes with results from published works and 3D-finite elements calculations made with the help of the user material subroutine (UMAT) in the FEM program ABAQUS. A comprehensive numerical analysis was conducted across a range of arrangements and thickness ratios to examine the impacts of viscoelasticity and inplane gradation and their combined effects on the hybrid laminated plates' free vibration behavior. The flexural frequency and related modes morphologies of the hybrid intelligent rectangular plate were found to be significantly altered by the in-plane gradation of stiffness and density. By choosing appropriate grading indices, the flexural frequency and stress within the plates could be altered. Furthermore, the electric behavior of the piezoelectric layer was far less affected by in-plane gradation, which was an intriguing finding as it can be crucial in the sensor design and actuator for dynamical use cases. Furthermore, based on the current viscoelastic modeling, the numerical research revealed a possible time-dependent structural behavior. Viscoelasticity analysis may be essential for a more realistic mechanical behavior analysis of various polymer composites and for future temporal programming in intelligent structure schemes that take advantage of the viscoelastic effect. Finally, it was possible to directly apply the current analytical solution to the analysis of both symmetric and asymmetric laminated piezoelectric smart plate with invariables attributes, even though it was initially suggested for the free vibration study of smart (IPFGMs) visco-elastic plate.

### 3. Conclusion

FGM orthotropic structures were widely applied in civil, mechanical, aeronautical, and aerospace

manufacturing due to their eye-catching features, such as high strength, outstanding stiffnesses, stiffness-weight ratios, lower cost, and high strength-to-weight ratio. The shear deformation influences were more noticeable in thick plate and shells. Accordingly, the vibration and instability examination of FGMs orthotropic plate and shells has been widely considered in recent years. Several procedures/approaches have been established by investigators for the examination of orthotropic FGM plates and shells.

The current manuscript survey the literature mostly for the linear and non-linear vibration and instability of orthotropic FGMs plate and shell. The theoretical and numerical procedures of solutions for the orthotropic FGMs structure are introduced.

The survey imposes the following remarks:

- a) More advanced shear deformation theories for plates and shells have been developed Because it is challenging to analyze FGMs orthotropic plates and shells using 2-D elasticity theory. These theories roughly approximate the 2-D elasticity solution with a good degree of precision. Thus, further work is needed in the future to develop revised theories that take transverse normal deformation into account.
- b) The nonhomogenous constraints of materials have a large outcome on dynamical stability and vibration response (critical loads, stability modes, deflections, linear and nonlinear frequencies). Furthermore, the outcome of nonhomogenous parameter for Young's modulus and shear moduli is highly remarkable than nonhomogenous parameter for density on structures behaviour. Hence, a rational designs of non-homogenous parameters is necessary for instability and vibration investigation for FGMs orthotropic plates and shells.
- c) Vibration and bucking of FGMs orthotropic plates and shells under constant, linear, and non-linear thermo-mechanical loads requires consideration in the future. The effect of temperature profile (heat conduction cases) on the thermo-mechanical behavior of FGMs orthotropic structure as unconventional shear deformation theories needs to be considered widely.

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## Conflicts of Interest

The authors declare no conflict of interest.

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